



Department of Mathematics and Statistics

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Math 117 - Spring 2023 - Common Final Exam, version A

Print name: _____

Section number: _____ **Instructor's name:** _____

Directions:

- Fill in your name and class section above, and keep this page closed until the start of the exam is called.
- This exam has 15 questions, worth a total of 120 points.
- Show your work. Answers (even correct ones) without the corresponding work will receive no credit.
- You may use a calculator that does not connect to the internet. The use of any notes or electronic devices other than a calculator is prohibited.

Good luck!

Question:	1	2	3	4	5	6	7	8
Points:	11	9	10	7	15	5	9	6
Score:								
Question:	9	10	11	12	13	14	15	Total
Points:	4	5	6	9	6	8	10	120
Score:								

Keep this page closed until the start of the exam is called.

Formulas

Average rate of change: $\frac{f(b) - f(a)}{b - a}$

Slope-intercept form: $y = b + mx$

Point-slope form: $y - y_0 = m(x - x_0)$

Standard form: $Ax + By = C$

Quadratic function: $y = ax^2 + bx + c$

Factored form: $y = a(x - r)(x - s)$

Quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Vertex form: $y = a(x - h)^2 + k$

Power function: $y = kx^p$

Directly proportional: $y = kx$

Inversely proportional: $y = \frac{k}{x}$

Factored form of a polynomial: $p(x) = c(x - a_1)(x - a_2) \cdots (x - a_n)$

1. Alice is deciding between two bids for a project. Both have fixed costs for materials and an hourly rate for labor. Company A has bid \$2000 for materials and \$60 per hour for labor. Company B has bid \$1200 for materials and \$80 per hour for labor.

(a) (4 points) Write a linear function for each company's bid as a function of t , the amount of labor required (in hours).

Company A: _____ Company B: _____

(b) (4 points) At what amount of labor time are the two bids equal? Give correct units for your answer.

(c) (3 points) For what amounts of labor time is Company A's bid cheaper than company B's bid? Give your answer using interval notation or inequalities, or in a sentence.

2. Let $P = f(t)$ be the population (in millions) of a country t years after 2020.

(a) (3 points) In a sentence with correct units, explain the meaning of $3.4 = f(4)$.

(b) (3 points) What are the units of the average rate of change $\frac{\Delta P}{\Delta t}$ for this function?

(b) _____

(c) (3 points) In a sentence with correct units, explain the meaning of $f^{-1}(7) = 8$.

3. The height (in feet) of an object after t seconds is given by

$$h(t) = -16(t - 2)^2 + 75.$$

(a) (4 points) Determine the maximum height of the ball and the time when that maximum occurs. Include correct units in your answer.

(b) (2 points) Give the equation of the line which is the axis of symmetry of the graph of $y = h(t)$.

(c) (4 points) Give the formula for the ball's height in standard form: $h(t) = at^2 + bt + c$.

Exam continues...

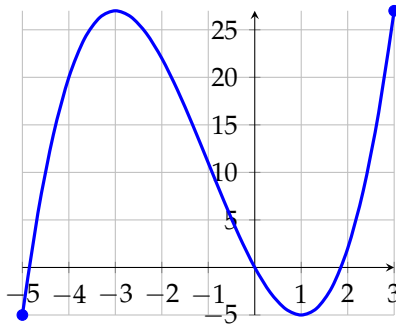
4. The table below gives some values of a function f and average rates of change $\frac{\Delta f}{\Delta x}$.

x	2	6	10	14
$f(x)$	20	18	14	2
$\frac{\Delta f}{\Delta x}$		$-\frac{1}{2}$?	-3

(a) (4 points) Find the missing rate of change in the table.

(b) (3 points) Is f concave up or concave down? Explain your answer in a sentence using the average rates of change in the table.

5. The graph $y = f(x)$ is given below. Use it to answer the following questions.



(a) (2 points) Estimate $f(1)$.

(a) _____

(b) (3 points) Estimate all solutions to $f(x) = 10$.

(b) _____

(c) (4 points) Estimate the average rate of change $\frac{\Delta f}{\Delta x}$ over the interval $-4 < x < 1$.

(d) (3 points) Over which intervals is f increasing? Circle all correct choices.

A. $-5 < x < -3$ B. $-3 < x < -2$ C. $-1 < x < 2$ D. $2 < x < 3$

(e) (3 points) Over which intervals is f concave up? Circle all correct choices.

A. $-5 < x < -4$ B. $-4 < x < -2$ C. $-1 < x < 2$ D. $2 < x < 3$

6. (5 points) Let $b(t) = t + 2$ and $r(t) = 3t^3$. Find a formula for the composition $b(r(t))$.

7. Suppose that $(4,7)$ is a point on the graph of f . Give a point on the graph of each of the following functions defined in terms of f .

(a) (3 points) $g(x) = 3f(x) - 2$

(a) _____

(b) (3 points) $h(x) = -f\left(\frac{1}{2}x\right)$

(b) _____

(c) (3 points) $h(x) = f(3(x+1)) - 5$

(c) _____

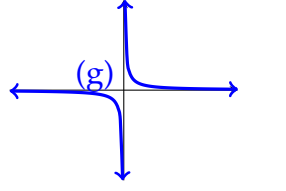
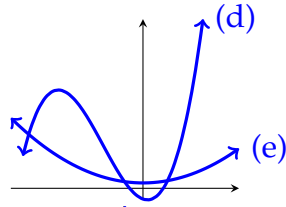
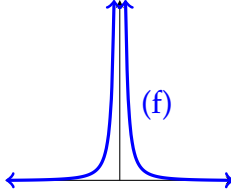
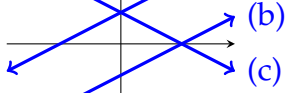
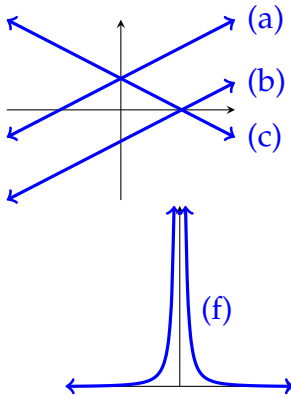
8. (6 points) Let $h(t) = \frac{3}{t-2}$. Find a formula for the inverse function h^{-1} .

9. (4 points) Find the domain of $g(x) = \frac{8}{\sqrt{x+7}}$. You may express your answer using interval notation, inequalities, or in a sentence.

Domain: _____

Exam continues...

10. (5 points) Match each of the following functions with a graph by writing the letter in the blank. The horizontal and vertical scales on the graphs may not be equal. Not all letters will be used.



_____ $y = \frac{10}{x}$

_____ $y = x^3 + 10x^2 - 8x - 20$

_____ $y = x^2 + 10$

_____ $y = 3x + 8$

_____ $y = 3x - 8$

11. Consider the rational function $y = r(x) = \frac{(x-2)(3x+1)}{(x-2)(x-4)}$.

(a) (3 points) Give the (x, y) coordinates of any holes in the graph of r .

(b) (3 points) Give the equation of any horizontal asymptotes of r .

12. Consider the rational function $y = g(x) = \frac{-3(x+2)}{(x+5)(x-\frac{2}{3})}$.

(a) (3 points) Give the equation of any vertical asymptotes of g .

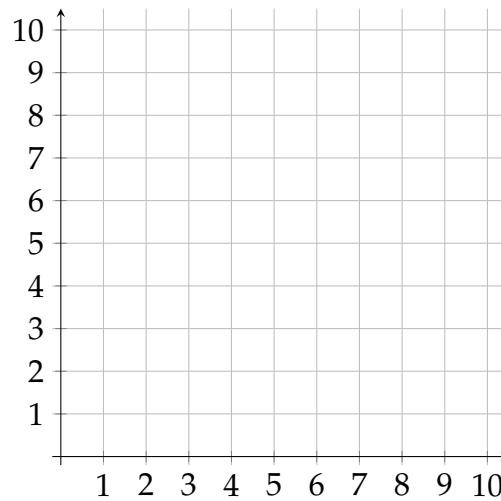
(b) (3 points) Give the x coordinate for any x -intercept of the graph of g .

(c) (3 points) Fill in the blanks with the correct end behavior of g .

$\lim_{x \rightarrow \infty} g(x) = \underline{\hspace{2cm}}$ and $\lim_{x \rightarrow -\infty} g(x) = \underline{\hspace{2cm}}$

13. (6 points) On the following set of axes, draw the graph of a function with these properties:

- The domain of the function is $0 \leq x \leq 8$.
- The range of the function is $1 \leq y \leq 10$.
- The function is increasing over its whole domain.
- The function is concave up for $0 < x < 4$ and concave down for $4 < x < 8$.



14. (8 points) Write a formula for a degree three polynomial with zeros at $x = -3$, $x = 4$, and $x = 5$, and which passes through the point $(0, 6)$.

15. Consider the function

$$f(x) = \begin{cases} -2x + 2 & \text{for } x \leq 0 \\ x^2 + 2 & \text{for } x > 0 \end{cases}.$$

(a) (3 points) Compute $f(4)$.

(b) (3 points) Find all solutions to $f(x) = 18$.

(c) (2 points) What is the domain of f ?

(c) _____

(d) (2 points) What is the range of f ?

(d) _____